arXiv:gr-qc/0210063v1 18 Oct 2002

DOUBLY-SPECIAL RELATIVITY: FIRST RESULTS AND KEY OPEN PROBLEMS

GIOVANNI AMELINO-CAMELIA

Dipartimento di Fisica, Universitá di Roma "La Sapienza", P.le Moro 2
00185 Roma, Italy

Received (received date)
Revised (revised date)

I examine the results obtained so far in exploring the recent proposal of theories of the relativistic transformations between inertial observers that involve both an observer-independent velocity scale and an observer-independent length/momentum scale. I also discuss what appear to be the key open issues for this research line.

1. Introduction

Over the last two years there has been a significant research effort aimed at the development of "Doubly-Special-Relativity" (or "DSR") theories [1], with contributions from about a dozen research groups [1−19]. These theories of the relativistic transformations between inertial observers involve both an observer-independent large-velocity scale and an observer-independent small-length/large-momentum scale. Some of the DSR results can already be seen as robust, and provide encouragement for the idea that DSR might plausibly have a role in quantum-gravity/quantum-spacetime research, which was my main motivation [1] in proposing the DSR research programme. As we start to establish some characteristic features of the DSR framework, we also start realizing that there are a few recurring themes of DSR research, issues that several authors have attempted to address, but still await a satisfactory analysis.

Here I intend to review the key results obtained so far in exploring the DSR framework, and I intend to discuss the key open issues for this research line. I will also present some preliminary ideas which might be useful in research concerning these open issues.

2. Doubly Special Relativity

In Galilei Relativity there is no observer-independent scale, and in fact (for example) the dispersion relation is written as $E = p^2/(2m)$ (whose structure fulfills the requirements of dimensional analysis without the need for dimensionful coefficients). As experimental evidence in favour of Maxwell equations started to grow, the fact

that those equations involve a fundamental velocity scale appeared to require (assuming the Galilei symmetry group should remain unaffected) the introduction of a preferred class of inertial observers (the "ether"). Einstein's Special Relativity introduced the first observer-independent relativistic scale (the velocity scale c), its dispersion relation takes the form $E^2 = c^2p^2 + c^4m^2$ (in which c plays a crucial role for what concerns dimensional analysis), and the presence of c in Maxwell's equations is now understood not as a manifestation of the existence of a preferred class of inertial observers but as a manifestation of the necessity to deform the Galilei transformations. The Galilei transformations would not leave invariant the relation $E^2 = c^2p^2 + c^4m^2$, which is instead an invariant according to the Lorentz transformations (the Lorentz transformations are a dimensionful deformation of the Galilei transformations).

I argued in Refs. [1] that it is not unplausible that we might be presently confronted with an analogous scenario. Research in quantum gravity*is increasingly providing reasons of interest in deformed dispersion relations of the general type $c^4m^2 = E^2 - c^2\vec{p}^2 + f(E, \vec{p}^2; E_p)$, where $E_p \sim 10^{28}eV$ is the Planck scale. Interest in these deformed dispersion relations is also coming as a result of the realization that they may provide a solution for the emerging GZK cosmic-ray anomaly (see below). The fact that these dispersion relations involve an absolute energy scale, E_p, was leading to the assumption that a preferred class of inertial observers might be introduced.

Similarly, deformed dispersion relations had also emerged in the mathematics programme of "quantum deformations" of classical algebras and groups. In particular, deformed dispersion relations had emerged in the study of the so-called κ-Poincaré Hopf algebras. Within some of these Hopf algebras there had been analyses of the action of exponentials of the Lorentz-like generatos, and on the basis of the results of these analyses (also see Section 8) it had been conjectured [29] that the actions described by exponentials of the κ-Poincaré Lorentz-like generators could not be combined to form a genuine group of transformations. This again suggested that deformed dispersion relations could not be implemented as genuine invariants of a (possibly quantum) group of Lorentz transformations.

In Refs. [1] I showed that it is incorrect to assume that a deformed dispersion relation necessarily implies the loss of covariance among inertial observers and the emergence of a preferred class of inertial observers. This observation has been then explored in detail in several studies [2−18]. Assuming for the sake of argument that a deformed dispersion relation was one day verified experimentally, we now know that this observation would not automatically imply that there is a preferred class of inertial observers. At least for certain choices of the function f in a dispersion relation of type $c^4m^2 = E^2 - c^2\vec{p}^2 + f(E, \vec{p}^2; E_p)$ it is possible to give to the deformed dispersion relation the same status that Special Relativity attributes to the dispersion relation $c^4m^2 = E^2 - c^2\vec{p}^2$. This however requires the introduction of one more (besides c) observer independent scale, E_p, and the structure of the boost transformations must be modified in such a way to leave invariant the deformed dispersion relation.

Concerning the choice of the deformed dispersion relation there appears to be at present a large level of uncertainty. To describe the situation by an analogy one should imagine attempts to introduce the first observer-independent relativistic scale, a velocity scale "c", without the clear experimental indication that c should be the speed of massless particles and the maximum attainable speed for massive particles. As discussed later in this note, there are some experimental contexts

*I am here thinking of some preliminary results in the study of Lie-algebra noncommutative spacetimes [20,21], the study of particle propagation [22,23] in the "Liouville-Noncritical-String" [24] model of spacetime foam, the study of loop-quantum-gravity scenarios [25,26,27], and the study of critical superstrings in an external B-field background [28].

which appear to invite us to consider a second observer-independent scale, some sort of high-energy scale, but these indications are only "tentative" and anyway they do not appear to single out a specific DSR theory.

In light of this situation, in proposing the DSR idea I found appropriate [1] to at least discuss an illustrative example of this new kind of relativistic theory. This example, now sometimes called "DSR1", has invariant dispersion relation

$$2E_p^2 \left[\cosh(\frac{E}{E_p}) - \cosh(\frac{m}{E_p})\right] = \vec{p}^2 e^{E/E_p} \ . \tag{1}$$

The DSR1 transformation rules, discussed in detail in Refs. [1,4,9], are very effectively characterized through the amount of rapidity needed to take a particle from its rest frame to a frame in which its energy is E (and its momentum is $p(E)$, which is fixed, once E is known, using the dispersion relation and the direction of the boost)

$$\cosh(\xi) = \frac{e^{E/E_p} - \cosh(m/E_p)}{\sinh(m/E_p)} \ , \quad \sinh(\xi) = \frac{p e^{E/E_p}}{E_p \sinh(m/E_p)} \ . \tag{2}$$

Another much studied example of DSR theory, which was proposed more recently by Maguejio and Smolin [5] and is sometimes called "DSR2", has dispersion relation

$$\frac{m^2}{(1-m/E_p)^2} = \frac{E^2 - p^2}{(1-E/E_p)^2} \ , \tag{3}$$

and it prescribes that the amount of rapidity needed to take a particle from its rest frame to a frame in which its energy is E should be given by

$$\cosh(\xi) = \frac{E(1-m/E_p)}{m(1-E/E_p)} \ , \quad \sinh(\xi) = \frac{p(1-m/E_p)}{m(1-E/E_p)} \ . \tag{4}$$

Other DSR theories are also being developed, but these first two illustrative examples remain the most studied ones.

3. What is DSR? What is not DSR?

Unfortunately, it is not uncommon to find in the physics literature a rather sloppy use of terms such as "fundamental scale". In particular, "fundamental scales" are often discussed as if they were all naturally described within a single category. For DSR research it is instead rather important that these concepts be introduced very carefully.

3.1. *c-type versus $\hbar$-type fundamental scales*

The DSR framework intends to introduce (at least) one more observer-independent relativistic scale. The prototype of relativistic scale is of course c. In the formulas reported in the preceding section the scale E_p is exactly on the same footing as c. But not all fundamental scales are introduced that way. A good counter-example is the quantum-mechanics scale $\hbar$.

Space-rotation symmetry is a classical continuous symmetry. One might, at first sight, be skeptical that some laws (quantum-mechanics laws) that discretize angular momentum could enjoy the continuous rotation symmetry, but more careful reasoning [30] will quickly lead to the conclusion that there is no *a priori* contradiction between discretization and a continuous symmetry. In fact, the type of discretization of angular momentum which emerges in ordinary non-relativistic quantum mechanics is fully consistent with classical space-rotation symmetry. All the measurements

that quantum mechanics still allows (a subset of the measurements allowed in classical mechanics) are still subject to the rules imposed by rotation symmetry. Certain measurements that are allowed in classical mechanics are no longer allowed in quantum mechanics, but of course those measurements cannot be used to characterize rotation symmetry (they are not measurements in which rotation symmetry fails, they are just measurements which cannot be done).

Just as in classical mechanics, when an observer O measures the square-modulus L^2 of the angular momentum, everything can be said about how that square-modulus appears to a second observer: the value of the modulus is the same for both observers. It happens to be the case that the values of L^2 are constrained by quantum mechanics on a discrete spectrum, but this of course does not represent an obstruction for the action of the continuous symmetry on invariants, such as L^2. When O measures exclusively the x component, L_x, of the angular momentum it is not possible to predict the value of any of the components of that angular momentum along the (x', y', z') axes of O'. This is true at the quantum level just as much as it is true at the classical level. This is another example of situation in which the fact that quantum mechanics constrains the values of an observable, L_x, on a discrete spectrum is irrelevant for rotation symmetry, since the relevant symmetry does not prescribe how that same observable is seen by another observer.[†]

A more detailed discussion of this point can be found in Ref. [30]. Essentially one finds that $\hbar$ is not a scale pertaining to the structure of the rotation transformations. The rotation transformations can be described without any reference to the scale $\hbar$. The scale $\hbar$ sets, for example, the minimum non-zero value of angular momentum ($L^2_{min} = 3\hbar^2/4$), but this is done in a way that does not require modification of the action of rotation transformations.

Galilei boosts are instead genuinely inconsistent with the introduction of c as observer-independent speed of massless particles (and maximum velocity attainable by massive particles). Lorentz transformations are genuinely different from Galilei transformations.

Both $\hbar$ and c are fundamental scales that establish properties of the results of the measurements of certain observables. In particular, $\hbar$ sets the minimum non-zero value of angular momentum and c sets the maximum value of speed. But $\hbar$ has no role in the structure of the transformation rules between observers, whereas the structure of the transformation rules between observers is affected by c. I am describing c as a relativistic fundamental scale, whereas $\hbar$ is a fundamental scale that does not affect the transformation rules between observers.

A characterizing feature of the DSR proposal is that there should be more than one scale playing a role analogous to c. In particular, the illustrative examples discussed above, DSR1 and DSR2, are relativistic theories with two observer-independent relativistic scales, the "speed-of-light" velocity scale c and Planck energy scale E_p.

One can try to introduce the Planck scale in analogy with $\hbar$ rather than with c. In particular, Snyder looked [31] for a theory in which spacetime coordinates would not commute, but Lorentz transformations would remain unmodified by the new commutation relations attributed to the coordinates. Such a scenario would of course not be a DSR.

In closing this subsection I should comment on one more type of fundamental constants. As I just explained, $\hbar$ and c are different types of fundamental constants but they both establish properties of the results of the measurements of certain observables. A third type of fundamental constants are the "coupling constants".

[†] Note that if another mechanical theory, clearly different from quantum mechanics, allowed simultaneous eigenstates of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ and predicted discrete spectra for all of them, then the classical continuous space-rotation symmetry would inevitably fail to apply.

For example, in our present description of physics the gravitational coupling G is a fundamental constant. It does not impose constraints on the measurements of a specific observable, but it governs the laws of dynamics for certain combinations of observables. Also G is observer indepedent, although a careful analysis (which goes beyond the scopes of this note) is needed to fully characterize this third type of fundamental scale. One can define G operatively through the measurement of static force between planets. In modern language this amounts to stating that we could define G operatively as the low-energy limit of the gravitational coupling constant. All observers would find the same value for this (dimensionful!) constant.

This last remark on the nature of the fundamental constant G is particularly important for DSR theories. In our present description of physics the Planck scale is just the square root of the inverse of G rescaled through $\hbar$ and c. The idea of changing the status of G (*i.e.* E_p) from the one of fundamental coupling scale to the one of relativistic fundamental scale should have deep implications [1,3].

In light of the preliminary success in contructing logically consistent DSR theories, one can imagine one day to devise a "triply-special" relativity in which also the status of $\hbar$ is changed to the one of relativistic fundamental scale.

3.2. *How many relativistic fundamental scales?*

Even within the DSR literature there has been some confusion about the concept of relativistic fundamental scale, and even on the counting of the relativistic fundamental scales present in a given DSR theory. From the equations (1),(2),(3),(4) it is clear that the theory has (at least in the energy-momentum sector, to which all robust results have been so far confined) two and only two relativistic fundamental scales.

The confusion has sometimes come from the fact that we are used to thinking interchangeably of energy and inverse length. One can of course think of the DSR deformation as a deformation involving the Planck length L_p rather than the Planck energy scale E_p, but this of course must (implicitly or explicitly) be understood in the sense that all energy-momentum space is described in terms of inverse lengths/times. Equivalently one can think of a DSR L_p-deformation as something which characterizes wavelength/frequency space and the laws of transformation for wavelengths and frequencies.

Instead some authors have taken literally/naively some DSR deformations as L_p-deformations of energy-momentum space, and they felt the need to introduce a third scale (perhaps identifiable with $\hbar$) to render dimensionless the product L_pE, between the Planck length and the energy of a particle being studied relativistically. This assumption is incorrect in the DSR theories discussed so far. There is no need for a third scale and if one naively introduced it by hand, in the way I just described, the scale would anyway not be a genuine relativistic scale, since it would not genuinely affect the structure of the laws of transformation between observers.

3.3. *Focus on the high-energy (high-momentum) regime*

Another main feature of the DSR proposal [1] is the objective of describing the short-distance/high-momentum regime. Just like Galilei Relativity becomes inadequate when velocities are high (comparable to the speed-of-light scale), the DSR research programme intends to explore the possibility that Einstein's Special Relativity might itself become inadequate in contexts in which ultrashort distances and/or ultrahigh single-particle momenta are involved. DSR's second relativistic observer-independent scale should become relevant only in short-distance/high-momentum regimes.

3.4. *Examples of the key role of the transformation rules: dispersion*

relation, maximum momentum, minimum-wavelength, and the number of relativistic scales

As already mentioned and discussed in the later sections of this note, there are some characteristic features that are common to most (if not all) DSR theories. Among these features much attention is directed in particular to the emergence of a deformed dispersion relation and the emergence of a maximum momentum/energy. One might be tempted to characterize a theory as DSR if it leads to these predictions, but actually a more careful attitude is necessary.

Of course the fact that a quantum gravity scenario predicts a Planck-scale deformation of the dispersion relation does not automatically imply that the laws of transformation between inertial observers be modified. In a theory that pefectly obeys the laws of ordinary Special Relativity one can of course find deformed dispersion relations in contexts in which there is some sort of background, a medium, a field distribution. And it is conceivable [23] that the structure of spacetime at short distances, often heuristically described in terms of a “spacetime foam”, might be described as a dispersion-inducing background.

Similarly the concept of a maximum momentum (and other similar concepts) does not automatically imply that the laws of transformation between inertial observers be modified. Rather than a reflection of deformed kinematics a maximum momentum could for example be a reflection of the structure of the laws of dynamics. Such a scenario has for example been explored in the context of string-bits models.

In some DSR approached one finds a maximu momentum and adopts a stadard relation between momentum and wavelength. As a result the maximum momentum corresponds to a minimum wavelength. Again this should be sharply distinguished from other quantum-gravity scenarios [32,33] in which one adopts a modified relation between momentum and wavelength, such that, although momentum is still allowed to go up to infinity (with boost transformations possibly described exactly as in ordinary Special Relativity), there is a finite minimum value of wavelength.

So, the issue of whether or not a given quantum-gravity scenario requires the DSR framework cannot be addressed at the level of the effects predicted within the physics-world picture of a single given observer: it requires an investigation of the transformation rules between inertial observers.

The transformation rules also define operatively the concept of a DSR theory. DSR does not intend to modify the operative concept of energy/momentum. We have adopted some “energy-momentum measuring devises” as the devises that provide an operative definition of energy-momentum. Procedures suitable for such a direct operative definition are for example in use at CERN and other particle-physics laboratory. If we put these CERN devises on a spaceship and compare the energy-momentum measurements reported by these spaceship devises to the same results obtained by identical devises placed on another spaceship, we presently predict that the measurement results on the two spaceships be connected by Special-Relativity transformations (in which the relative velocity of the spaceships place a key role, which is fully specified by Special Relativity). This comparison of measurement results is a well-defined operative procedure for wich Special Relativity makes definite predictions and DSR theories will make alternative predictions. The issue can therefore at least in principle (and, as I will emphasize later, in some cases even in practice) be settled experimentally.

The fact that one can introduce formal nonlinear maps between different DSR theories has led some authors [8] into the erroneous conclusion that these theories might be equivalent. Of course, the existence of some nonlinear maps that connect different DSR theories, does not imply the physical equivalence of the theories: on the contrary it simply establishes the relations between the <u>different</u> physical predictions of different DSR theories. A possible source of this confusion may come from

the fact that those working in the DSR framework often have a formal General-Relativity background. In General Relativity even a nonlinear map between spacetimes may lead to physically equivalent spacetimes if corresponding changes are implemented on the metric tensor. This is the core ingredient of the diffeomorphism invariance of General Relativity. Spacetime coordinates are not themselves observable. Distances (a concept which necessarily involves the metric tensor) between spacetime point are observable, but the spacetime coordinates are not themselves observable. But these nonlinear maps that connect DSR theories are maps between energy-momentum spaces, and energy/momentum are directly observable. There is no diffeomorphism invariance of energy-momentum space. Different DSR theories are therefore clearly inequivalent, and in fact, as discussed in greater detail later in these notes, they actually lead to different predictions for certain classes of experiments, so we have identified physical contexts in which these different theories give rise to explicitly different physical predictions (and this of course excludes the physical equivalence of the theories).

4. Doubly-Special-Relativity theories: DSR1, DSR2 and DSR3

Up to this point I have explicitly described only the first two illustrative examples of DSR theories, DSR1 and DSR2. Each of these two examples is actually a good representative of a corresponding class of DSR theories, which I will propose to call DSR1-type theories and DSR2-type theories.

In introducing these classes of DSR theories I want to make reference to their key physical predictions. There has been extensive discussion in the recent quantum-gravity literature of the fact that there are two classes of observations which have extremely high sensitivity to possible Planck-scale deformations of kinematics. The first context that deserves mention is the one of experiments looking for possible Planck-scale-induced wavelength-dependent relative delays in the times of arrival of (nearly-) simultaneously emitted photons. In certain astrophysical contexts [23,34,35] this effect can be investigated with very high sensitivity. A second relevant context is the one of the threshold conditions for particle production in certain collision processes. When one of the colliding particles has ultrahigh energy and the other particle is a very soft photon (a situation which is relevant in certain astrophysical contexts, such as the GZK limit for cosmic-ray physics), certain Planck-scale deformations of special-relativistic kinematics lead to effects that are observably large, particularly through their implications [36,37,38] for the observations of ultra-high-energy cosmic rays.

The DSR1 theory is a theory in which, for particles with $E \ll E_p$, there is indeed a wavelength dependence of the speed of photons, and this effect comes in at the E/E_p level, as the reader can easily verify applying the relation $v = dE/dp$ to the DSR1 dispersion relation. This wavelength dependence at the level E/E_p can be tested with forthcoming experiments [23,34,35]. Instead DSR1 does not predict [1,3] an observably large modification of the threshold conditions for particle production in collision processes.

In the DSR2 theory the dispersion relation for photons ($m = 0$) is unmodified. There is thefore no wavelength dependence of the speed of photons. In addition also the DSR2 theory does not predict [6] an observably large modification of the threshold conditions for particle production in collision processes.

A DSR1-type theory will be any DSR theory in which the wavelength dependence of the speed of photons comes in at the level E/E_p, and there is no observably-large modification of the threshold conditions for particle production in collision processes.

A DSR2-type theory will be any DSR theory in which the wavelength dependence of the speed of photons comes in at the level $(E/E_p)^\alpha$, with $\alpha \geq 2$ (in particular DSR2 has $\alpha = \infty$) and there is no observably-large modification of the

threshold conditions for particle production in collision processes.

I recently proposed [18] a third type of DSR theory, DSR3-type theories, which is based on the presence of a specific structure in the deformation of Special Relativity that DSR implements. In ordinary Special Relativity the amount of rapidity needed to take a particle from its rest frame to a frame in which it has energy E is $\cosh(\xi) = E/m$. In DSR1 and DSR2 this relation is modified (see Eqs. (2),(4)) and it is noticeable that the modifications are structured in terms of the quantities E/E_p and m/E_p, which appear separately in the deformation. I Ref. [18] I argued that interest may be deserved by DSR theories in which this deformation attributes a key role to the quantity $E^2/(mE_p)$. As an illustrative example of the way in which the "$\cosh(\xi)$ relation" could be modified I considered the case

$$\cosh(\xi) = \frac{E}{m}(2\pi)^{-E^2 \tanh[m^2 E_p^4/E^6]/(mE_p+E^2)} \ . \tag{5}$$

The reader should not be deterred by the apparently *ad hoc* form of Eq. (5). We clearly do not yet have a compelling DSR3-type theory, but relation (5) can be used to illustrate some of the compelling features that these theories can have.

A DSR3-type theory will be a theory in which the "$\cosh(\xi)$ relation" is modified in a way that attributes a key role to the quantity $E^2/(mE_p)$. DSR3-type theories may or may not have observably-large wavelength dependence of the speed of photons (this depends on the full structure of the theory, which is not specified by just giving the "$\cosh(\xi)$ relation"). DSR3-type theories will be characterized by important effects in the regime in which the energy of the particle is in the neighborhood of the value $\sqrt{mE_p}$. This may be particularly significant in the study of modifications of the threshold conditions for particle production in collision processes. In particular the derivation of the GZK limit involves a proton of energy $\sim 10^{19} eV$, and for a proton the quantity $\sqrt{mE_p}$ takes the value $\sim 3{\cdot}10^{18} eV$. Therefore the implications for the GZK limit are naturally going to be observably large in DSR3-type theories.

5. Brief tutorial on the construction of DSR theories

Various strategies have been exploited in constructing "DSR theories" (DSR-type laws of transformation between inertial observers). In formulating the DSR proposal I thought [1] it would be safest to take a starting point "*a la* Einstein": introducing some postulates that were directly associated with the relativistic observer-independent scales and deriving from those postulates the structure of the theory. I believe that this remains the most "physical" strategy, but other strategies appear to lead to equally-robust results.

While a careful examination of the DSR literature is of course the best way to become familiar with relevant techniques, here I want to describe briefly some lines of analysis that apply to (what appears to be) the simplest class of DSR theories: theories based on a deformed dispersion relation and in which a key role is played by a nonlinear realization of the Lorentz symmetry group. I take as illustrative example the DSR1 theory.

In constructing DSR1 one can start from the dispersion relation (which could, for example, be the result of an experimental analysis)

$$2E_p^2 \left[\cosh(\frac{E}{E_p}) - \cosh(\frac{m}{E_p})\right] = \vec{p}^2 e^{E/E_p} \ . \tag{6}$$

This dispersion relation is clearly an invariant of space rotations, but it is not an invariant of ordinary boost transformations. The next step is the one of finding

deformed boost transformations which have (6) as an invariant. In ordinary Special Relativity the boosts can be described by

$$B_a = ip_a \frac{\partial}{\partial E} + iE \frac{\partial}{\partial p_a} \ , \tag{7}$$

and in the DSR1 case one can make the ansatz

$$\mathcal{B}_a = i\Delta_1(E, p^2, E_p) p_a \frac{\partial}{\partial E} + i\Delta_2(E, p^2, E_p) \frac{\partial}{\partial p_a} + i\Delta_3(E, p^2, E_p) p_a p_b \frac{\partial}{\partial p_b} \ , \tag{8}$$

which is already aiming at automatically preserving space-rotation symmetry.

Demanding that (6) is an invariant of $\mathcal{B}_a$ transformations already provides a first condition on Δ_1, Δ_2 and Δ_3. Next one should investigate the commutators of the $\mathcal{B}_a$ generators with the undeformed space-rotation generators

$$R_a = -i\epsilon_{abc} p_b \frac{\partial}{\partial p_c} \ . \tag{9}$$

The six generators $\mathcal{B}_a$, R_a should close an algebra, and after some thinking one can easily conclude that this algebra must still be the usual Lorentz algebra. In fact, the operators $\mathcal{B}_a$ and R_a have the same dimensions (in the sense of elementary dimensional analysis) and this forbids the introduction of the deformation scale E_p in the commutation relations that define the algebra. One could perhaps consider an extension of the Lorentz algebra: replacing the six-generator Lorentz algebra with a larger algebra. For example, one could postulate commutators of the rotation/boost generators that depend also on momenta. But this would result in a theory in which finite rotation/boost transformations do not really form a group: the way in which combinations of rotation/boost transformations act on a given momentum would depend on the value of that momentum, rather than being an intrinsic property of the given combination of rotation/boost transformations. [For example, in ordinary Special Relativity the combination of two boosts can give a rotation, and this rotation has no dependence on the momentum on which the two boosts are applied. This can be shown to be directly connected with the fundamental concept of spacetime symmetry.] It appears reasonable to reject‡ this type of pathology of rotation/boost transformations, and this leads to the only option of maintaining the Lorentz algebra. In spite of the deformation of the $\mathcal{B}_a$ generators, the six generators $\mathcal{B}_a$, R_a must close the usual Lorentz algebra. This leads to other conditions on the Δ_1, Δ_2 and Δ_3.

Combining the conditions coming from the invariance of the dispersion relation and the Lorentz-algebra conditions one can determine (up to some arguments of "simplicity": Δ_1, Δ_2 and Δ_3 are not [39] fully fixed by the conditions, but they are indeed fixed if they are assumed to have relatively simple mathematical structure) the full structure of the deformed boost generators. In the DSR1 case one finds:

$$\mathcal{B}_a = ip_a \frac{\partial}{\partial E} + i\left(\frac{1}{2E_p}\vec{p}^2 + E_p \frac{1 - e^{-2E/E_p}}{2}\right) \frac{\partial}{\partial p_a} - i\frac{p_a}{E_p}\left(p_b \frac{\partial}{\partial p_b}\right) \ . \tag{10}$$

It is useful to obtain explicit formulas for the finite boost transformations that relate the observations of two observers. These are obtained by integrating the

‡There is a strong connection between this argument for rejecting deformations of the Lorentz algebra and the problems which had been encountered in the κ-Poincaré literature in relation with the puzzling emergence of a "quasi-group" structure (see Section 9).

familiar differential equations

$$\frac{dE}{d\xi} = i[\mathcal{B}_a, E] \ , \quad \frac{dp_b}{d\xi} = i[\mathcal{B}_a, p_b] \ , \tag{11}$$

which relate the variations of energy-moomentum with rapidity (ξ) to the commutators between the boost generator and energy-momentum (and of course these commutators are implicitly coded in (10)).

In spite of the richer structure of the deformed boost generators, the derivation of finite transformations from the structure of the generators of infinitesimal transformations is rather straightforward [1,4], and can be done in full generality. Among the results that provide most insight in the structure of the DSR theory are the mentioned ones that specify the amount of rapidity needed to take a particle from its rest frame to a frame in which its energy is E. As already noted above, in the case of DSR1 one finds

$$\cosh(\xi) = \frac{e^{E/E_p} - \cosh(m/E_p)}{\sinh(m/E_p)} \ , \quad \sinh(\xi) = \frac{p e^{E/E_p}}{E_p \sinh(m/E_p)} \ . \tag{12}$$

Relations such as these (12) can actually be taken as starting point for the construction of a DSR theory based on a nonlinear realization of the Lorentz group. The underlying structure of a nonlinear realization of the Lorentz group implies that relations such as (12) should be obtainable through a nonlinear (and, as mentioned, physically inequivalent) redefinition of the energy-momentum variables. In ordinary Special Relativity (governed by the linearly-realized Lorentz group) the relations (12) take the form

$$\cosh(\xi) = \frac{\epsilon}{\mu} \ , \quad \sinh(\xi) = \frac{\pi}{\mu} \ , \tag{13}$$

where I adopted the notation ϵ,π for energy-momentum variables that transform according to ordinary Special Relativity, and I introduced the invariant $\mu \equiv \sqrt{\epsilon^2 - \pi^2}$. The role played in DSR1 by a nonlinear realization of the Lorentz group implies that it should be possible to introduce throughout the theory some special combinations of the DSR1-physical energy-momentum variables E,p that transform instead linearly under the DSR1 boosts. By comparison of (12) and (13) one easily identifies these special functions of the DSR1-physical energy-momentum variables E,p:

$$\frac{\epsilon(E, m; E_p)}{\mu(m)} = \frac{e^{E/E_p} - \cosh(m/E_p)}{\sinh(m/E_p)} \ , \quad \frac{\pi(p, E, m; E_p)}{\mu(m)} = \frac{p e^{E/E_p}}{E_p \sinh(m/E_p)} \ . \tag{14}$$

The function $\mu(m)$ is obtained from the condition

$$\mu(m)^2 = \lim_{p \rightarrow 0 \ , \ E \rightarrow m} \left[\epsilon(E, m; E_p)^2 - \pi(p, E, m; E_p)^2 \right] \ , \tag{15}$$

and then the functions $\epsilon(E, m; E_p)$ and $\pi(p, E, m; E_p)$ are fully specified by (14).

Similar relations can be found with analogous reasoning in all DSR theories (with underlying nonlinear realization of Lorentz symmetry). As mentioned, these relations of the type (14) can provide the starting ingredient for the construction of a DSR theory. Taking as starting point the functions $\epsilon(E, m; E_p)$ and $\pi(p, E, m; E_p)$ (and the function $\mu(m)$ associated to them through (15)) one immediately has an (implicit) description of the transformation rules, and the $E(p)$ dispersion relation automatically takes the form

$$\mu(m)^2 = \epsilon(E, m; E_p)^2 - \pi(p, E, m; E_p)^2 \ . \tag{16}$$

The deformed boost generators can then be derived from the structure of this dispersion relation or simply using

$$\mathcal{B}_a = i\pi_a \frac{\partial}{\partial \epsilon} + i\epsilon \frac{\partial}{\partial \pi_a} \tag{17}$$

(in which ϵ and π_a are understood as functions of the DSR-physical energy-momentum E,p_a).

6. First results

6.1. *Wavelength-dependent speed of photons and threshold conditions in collisions*

As already mentioned above, the debate on DSR theories has correctly kept in focus the issue of the predictions concerning a possible wavelength dependence of the speed of photons and a possible modification of the threshold conditions for particle production in collision processes. In fact, these are key aspects of relativistic kinematics, and there are some chances of obtaining encouragement from planned experiments.

6.2. *Transformation rules (one-particle case)*

For DSR1 and DSR2 there has been explicit detailed analysis of the laws of transformation of energy and momentum from one inertial observer to another. This has also allowed to develop techniques of analysis which are very powerful and appear to be applicable to a relatively large class of DSR theories (although no other example of DSR theory has been analyzed in detail).

6.3. *Maximum momentum*

As already expected upon proposing the DSR framework [1], it has been fully established that DSR theories may provide a natural framework in which to implement the quantum-gravity idea of a maximum momentum and/or energy. This result is obtained kinematically. For example in DSR1 the laws of transformation between inertial observers [1,4] are such that any given momentum can be maximally boosted to the value E_p/c, where E_p is the observer-independent relativistic deformation scale (naturally identified, up to a coefficient not-too-different from 1, with the Planck scale). In DSR2 both energy and momentum can only be maximally boosted to maximum values set by E_p and E_p/c respectively.

6.4. *Laws of composition of energy-momentum*

As also already expected upon proposing the DSR framework [1], it has been fully established that DSR theories require a nonlinear law of composition of energy and momentum. This property can be understood in analogy with the fate of the law of composition of velocities in going from Galilei Relativity to Einstein's Special Relativity. In Galilei Relativity there is no absolute velocity scale and, as a consequence, the law of composition of velocities could not be anything else but linear. A linear law of composition of velocities is of course incompatible with the existence of a maximum velocity. Special Relativity required a nonlinear law of composition of velocities. Just like the Galileian linear law of composition of velocities must be rejected upon introducing an absolute velocity scale, the special-relativistic law that composes linearly the energies and momenta of particles must be rejected [1] upon introducing an absolute energy/momentum scale.

For DSR1 and DSR2 acceptable laws of composition of energy-momentum (laws with the needed covariance properties under DSR transformations) have been constructed.

6.5. *Deformed Klein-Gordon/Dirac/Maxwell equations*

The first steps have recently been taken toward the construction of theories that are consistent with DSR kinematics. This programme has taken has starting point the structure of the deformation of the Klein-Gordon, Dirac and Maxwell equations in energy-momentum space.

The Klein-Gordon equation in energy-momentum space is basically a direct reflection of the dispersion relation, so it can be constructed straightforwardly in DSR theories which have been analyzed at the level of the dispersion relation, such as DSR1 and DSR2.

In work (that progressed in parallel and interconnectedly) by Arzano and myself [10] and by Ahluwalia and Kirchbach [11] the structure of the Dirac equation for DSR1 and DSR2 in energy-momentum space has been established. In particular for DSR1 this deformed Dirac equation can be conveniently written as

$$(\gamma^\mu \mathcal{D}_\mu(E,p,m;E_p) - I)\,\psi(\overrightarrow{p}) = 0 \tag{18}$$

where

$$\mathcal{D}_0 = \frac{e^{E/E_p} - \cosh{(m/E_p)}}{\sinh{(m/E_p)}}\ , \tag{19}$$

$$\mathcal{D}_a = \frac{p_a}{p}\frac{\left(2e^{E/E_p}\left[\cosh{(E/E_p)} - \cosh{(m/E_p)}\right]\right)^{\frac{1}{2}}}{\sinh{(m/E_p)}}\ , \tag{20}$$

and the γ^μ are the familiar "γ matrices".

There appears to be no in-principle problem in writing analogously a Maxwell equation, but this has not yet been done.

7. Open problems for DSR

7.1. *How should one describe macroscopic bodies in DSR?*

One key issue for the DSR research programme is the one concerning the description of macroscopic bodies. Planck-scale deformations of the dispersion relation are clearly admissable for microscopic particles, since we produce/observe these particles with energies that are much smaller than E_p and therefore the predicted new effects are small enough to comply with present experimental limits. Instead macroscopic bodies typically have energies largely in excess of E_p, and the assumption of a Planck-scale deformation of the dispersion relation is in clear conflict with observations as ordinary as the motion of planets in the solar system and the game of soccer.

It has been clear from the beginning [1] that the DSR framework automatically includes structures that may be useful in settling satisfactorily this issue, but a full satisfactory understanding/description is still missing.

7.2. *What is the DSR observer?*

An issue which is possibly related with the above-mentioned issue concerning macroscopic bodies is the one that concerns the description of DSR observers. It appears likely that the theory will require the obsever to be a macroscopic system, to which the Planck-scale deformation does not apply. Since we lack a description

of macroscopic bodies, we are also still lacking a genuine understanding of DSR obsevers.

It also appears that one should think of these observers as equipped with a large variety of probes. Since the speed of photons is (in generic DSR theories, not in DSR2) energy dependent, some of the perceptions of the observer should perhaps depend on the type of probes the observer uses in a given context. For example, in a context probed with high-energy probes the observer might experience a different type of time dilatation and leangth contraction. But this has not yet been investigated. Perhaps, rather than thinking of a single observer with different types of probes, we should think of different types of observers, characterized by the type of probes they use.

At the merely technical level these issues concerning the DSR observer are relevant for establishing the relation between rapidity and relative velocity among observers. Is this relation modified? Or should we still adopt the special-relativistic relation between rapidity and relative velocity?

7.3. *Which spacetimes are compatible with the DSR framework?*

All robust results obtained so far in the DSR framework concern the energy-momentum sector. It is unclear which type of spacetime picture is required by the DSR framework. We should be prepared for a significant "revolution" in the description of spacetime. The introduction of the first relativistic observer-independent scale, c, forced us to renounce to the (until then unquestioned) concept of absolute time. What should we give up for the second observer-independent scale?

It appears reasonable (though not the only plausible choice) to insist on a spacetime picture which is consistent with the relation $v = dE/dp$ between the velocity of a particle and its energy-momentum. This relation holds in Galilei relativity, and it turned out to survive the introduction of the first obsever-independent scale in Special relativity, so it appears natural to assume that it would survive also the advent of a second observer-independent scale. This relation also means that the relativistic theory is consistent with some kind of Hamiltonian dynamics: $dx/dt = dH/dp$.

If $v = dE/dp$ should indeed be enforced, the implications for the spacetime picture could be profound. In typical DSR theories, with their key nonlinearities, one observer, O, could see two particles with different masses m_A and m_B moving at the same speed and following the same trajectory (for O particles A and B are "near" at all times), but the same two particles would have different velocities according to a second observer O', so they could be "near" only for a limited amount of time. For the particles we are able to study/observe, whose energies are much smaller than the Planck scale, and for the type of (relatively small) boosts we are able to investigate experimentally, this effect can be safely neglected. But conceptually it has striking implications. The possibility of the absolute statement "particles A and B follow the same trajectory" would be removed from our spacetime picture.

Perhaps this is taking us in the direction of considering spacetime as an approximate concept, only valid within a certain class of observations and with a certain level of approximation. Perhaps such a picture could be implemented through a description of spacetime in terms of noncommutative geometry. Noticeably, there is at least one noncommutative spacetime in which the DSR framework appears to be applicable. This is the "κ-Minkowski" spacetime [40,20]

$$[x_j, t] = i\lambda x_j \ , \quad [x_j, x_k] = 0 \ , \tag{21}$$

where the product of plane waves has the property

$$\left(e^{ip_m x_m} e^{ip_0 x_0}\right)\left(e^{ik_m x_m} e^{ik_0 x_0}\right) = e^{i(p_m + e^{\lambda p_0} k_m) x_m} e^{i(p_0 + k_0) x_0} \tag{22}$$

Eq. (22) clearly relfects a nonlinear law of composition of momenta and is therefore consistent with the basic structure of DSR theories.

7.4. *Can DSR solve the GZK anomaly?*

As mentioned the DSR framework naturally leads to the prediction of modified threshold conditions for particle production in certain collision processes. This can be perceived as an opportunity for DSR: in fact, in astrophysics there has been much recent discussion [36,38,41] of anomalies possibly related with such threshold conditions. In particular, the analysis of a threshold condition is key for the GZK limit on the observations of cosmic rays. Cosmic rays travel toward us in the environment of the CMBR photons. When the energy of the cosmic ray exceeds the threshold value $5{\cdot}10^{19}eV$, according to ordinary Special Relativity it should be possible for the cosmic ray to loose energy through pion production off CMBR photons. This should render observations of cosmic rays above $5{\cdot}10^{19}eV$, the GZK limit, extremely unlikely. Still, more than a dozen UHECRs have been reported by AGASA [42] with nominal energies at or above 10^{20} eV.

While DSR theories do generically predict anomalous thresholds (and therefore generically predict a different value of the GZK limit), as mentioned in DSR1 and DSR2 the change in the value of the GZK limit is very small, and could not be used to explain away the paradoxical ultra-high-energy cosmic-ray observations.

Even looking beyond DSR1 and DSR2 it appears hard[§] to find a DSR theory which would significantly affect the GZK limit.

The AGASA evidence of a cosmic-ray paradox must be considered as preliminary. Forthcoming more accurate cosmic-ray observations [43] may well show us[¶] that there is no cosmic-ray paradox after all. Still, it is frustrating that one of the few cases in which ordinary Special Relativity is being justifiably questioned appears to be also a context in which the predictions of DSR theories do not differ significantly from the ones of Special Relativity.

7.5. *What about causality?*

In a significant portion of DSR theories, including DSR1, there appear to be also profound consequences for causality. These theories predict that ultra-high-energy photons would travel faster than the low-energy photons which we ordinarily observe/study (for which it is well established that they travel at speed c). Actually, in DSR1 the limit in which a single particle has infinite energy is also the limit in which the speed of that particle is itself infinite. Of course, we cannot even contemplate a particle with infinite energy (we can at best, very optimistically of course, contemplate the possibility to put all the energy on the Universe in a single particle), but nevertheless it is natural to aspect that these properties would have profound implications for our understanding of causality.

The implications can be positive: for example, some authors have used (see, *e.g.*, Ref. [45]) some preliminary intuition about the new causality to construct cosmological models based on DSR that would not require inflation. These proposals

[§]Some authors [16] have given up so completely on the hope of finding a DSR solution for the cosmic-ray puzzle that they arrived at proposing that solutions of the cosmic-ray puzzle would necessarily require <u>two</u> length/energy scales, rather than the single one available in DSR theories.

[¶]It is not uncommon that preliminary data generate interest in related theory subjects, and in some cases the lessons learned through those theoretical studies outlast the possible negative evolution of the experimental situation. This author is familiar [44] with the theory work that was motivated by the so-called "centauro events". It is now widely believed that centauro events were a "mirage", but in the process we did learn that the formal structure of QCD allows the vacuum to be temporarily misaligned (disoriented chiral condensates) and the RHIC collider is conducting dedicated experiments. The cosmic-ray paradox might or might not disappear when more accurate and reliable data become available; however, in either case, the debate on the cosmic-ray paradox has had the merit to push us to discover that certain types of deviations from ordinary Lorentz symmetry are plausible in quantum-gravity scenarios.

provide an elegant reinterpretation of "varying-speed-of-light cosmology" [46,47,48]: rather than assuming an explicit time dependence of the speed of photons (which would inevitably lead to a breakup of Lorentz symmetry), one works within a DSR framework *a la* DSR1, in which the speed of photons increases with energy, and then observes that in the Early Universe particles typically had very high energies and (within DSR1) could put in causal connection regions of the cosmo which instead would be causally disconnected according to ordinary Special Relativity.

But a full understanding of DSR causality is clearly a key objective of this research programme. We should perhaps classify inertial observers in classes depending on the probes they have available. Would the observers which have ultra-high-energy photons at their disposal be able to introduce a concept of time that is (to good approximation) absolute? The dilatation of the muon lifetime due to its velocity would depend on which probes we use to establish this lifetime?

7.6. *What about noninertial observers?*

Of course, since the DSR research programme is considering a modification of Special Relativity, a natural next step to consider is the one in which an analogous modification is implemented at the level of General Relativity. Nothing that can claim any robusteness has been obtained so far on this interesting point. I believe [3] that a key issue for such studies comes from the observation that we might be required to attribute to the Planck scale a double role: a role in the gravitational coupling (because of the relation between G and E_p) and a role in the structure of spacetime (energy-momentum space). If this intuition turns out to be correct we might have to face significant challenges at the conceptual level. It is always very significant when two operatively well-defined concepts turn out to be identified (see, *e.g.*, the Equivalence Principle for inertial and gravitational mass).

8. Some ideas for the open problems

As I was discussing the "open problems" for the DSR research programme I already mentioned a few ideas that may prove useful, as in the case of inertial observers classified on the basis of the typical energies of the probes available to them. For some other ideas I thought it might be appropriate to have this dedicated section.

8.1. *Macroscopic bodies severely affected by the nonlinearity of DSR theories*

The fact that ordinary Special Relativity has an absolute velocity scale forced to compose velocities nonlinearly. We compose velocities only in one context: when we compare the velocity that a particle has for a given observer O with the velocity that the same particle has for another observer O' (itself moving at some velocity with respect to O). In DSR theories there is also a large-energy/small-length scale and this of course imposes that energy-momentum be composed nonlinearly. We clearly need to compose energy-momentum if we want to impose some sort of conservation of "total" [1,3] energy-momentum in collision processes. There is also another context in which we, in a certain sense, compose energy-momentum: a macroscopic body is "made of" a large number of microscopic particles, and its own energy-momentum is obtained "composing" the energy-momentum of the microscopic particles it is made of and taking into account binding energies.

When we compose energy-momentum in the context of conservation laws for particle-production processes we are genuinely dealing with pure kinematics. But when we describe a macroscopic body in terms of the microscopic particles it is made of it is necessary to consider also dynamics. Perhaps this description of macroscopic bodies (for which we are presently not ready) will provide the answers we are seeking.

One can even speculate that through the mechanism that allows a macroscopic body to be formed out of many microscopic particles the DSR effects might be screened, just like in quantum mechanics the quantum properties of microscopic particles are screened in cases in which these microscopic particles are "put together" into a macroscopic body. The analogy with quantum mechanics finds also some intuitive (though not yet robust) support in the fact that the Planck scale does involve Plack's constant $\hbar$.

8.2. *Macroscopic bodies made of particles with minimum wavelength*

Perhaps instead the clarification of the "macroscopic-body problem" will come from other mechanisms. Perhaps DSR should be used to describe a minimum wavelength rather than a maximum momentum. This would at least provide an acceptable description of beams of particles. A laser beam of photons has much larger momentum that the momentum of its composing photons, but both the beam and each of the composing particles have the same wavelength. A deformed wavelength/frequency dispersion relation is acceptable for both the beam and the single photon, while a deformed energy/momentum dispersion relation cannot be applicable to the macroscopic beam.

A beam of particles is much simpler than a macroscopic body (in which microscopic particles are bound), but also for a macroscopic body a deformed wavelength/frequency relation coudl differ from a deformed energy/momentum relation in a significant way.

8.3. *Macroscopic bodies a la DSR3*

Perhaps the solution of the "macroscopic-body problem" is even simpler. In DSR theories of type DSR3 one can easily obtain that the effect is automatically confined to particles with small mass. In particular, a DSR deformation of Special Relativity based on a relation of the type (5) could be applied directly to macroscopic bodies, since it leads to vanishingly small effects in the large-mass limit. Moreover, in spite of the $E^2/(mE_p)$ dependence, the relation (5) is structured in such a way that it also leads to the absence of any deformation for massless particles. The DSR effects suggested by the DSR3-type relation (5) are significant only for particles with small mass m (much smaller than E_p/c^2) and only when these particles have energies in the neighborhood of $\sqrt{mE_p}$.

8.4. *GZK anomaly a la DSR3*

The DSR3-type theories which I proposed to consider in Ref. [18] might also provide a straightforward solution for the cosmic-ray paradox. As shown in Ref. [18], contrary to DSR1 and DSR2, these DSR3-type theories can lead to a significant shift of the GZK limit.

8.5. *GZK anomaly as a reflection of the properties of composite particles*

The DSR framework naturally leads [1,3] to the concept of a fundamental building block for particles. In fact, DSR usually requires different kinematical properties for composite particles (up to macroscopic bodies) and fundamental particles. For the first time in the history of physics there might be a meaningful (rather than phylosophical) issue concerning the identification of fundamental particles. Which particles are fundamental?

The GZK limit for cosmic-ray observations crucially depends [36,38] on the analysis of photopion production $p+\gamma \rightarrow p+\pi$ (incoming proton and photon, outgoing proton and pion). Should we apply the charateristic dispersion relation of a given DSR to

pions and protons? Or should we describe pions and protons as some sort of DSR bound state of quarks (which, as a DSR multiparticle body, might obey a different dispersion relation)? Should we even think of quarks and photons as fundamental? An ordinary photon is huge with respect to the Planck length!

The DSR analysis of the GZK limit might therefore be highly nontrivial. It might be [12] connected with the problematic description of bound systems and macroscopic bodies.

Incidentally, let me observe that this perspective on the cosmic-ray paradox is also connected with the questions about a DSR spacetime, mentioned in Subsection 6.3. If we allow different particles (protons, pions, photons...) to obey different kinematic laws, then the scenario in which spacetime is a derived approximate concept appears to become inevitable. One could abstract a spacetime in contexts (the only contexts to which we presently have access) in which all particles have energy/momentum much smaller than the Planck scale, but in contexts in which particles with energy/momentum comparable to the Planck scale are available (like the early Universe) it would not even be possible to introduce a good approximate concept of spacetime.

9. Comparison with previous literature

The DSR literature has grown rapidly in this first two years of existence. Many robust results have been obtained. Naturally, some incorrect descriptions have also surfaced in the DSR literature. In particular, there are very different views on the connection between DSR theories and other ideas for departures from Special Relativity which have also been considered in the literature.

An analysis of research developments over the last few years shows that the true root of the DSR proposal put forward in Refs. [1] is in the previous studies [23,25] which were based on the idea of spacetime-foam-induced deformed dispersion relations. Those spacetime-foam scenarios require that Lorentz symmetry be broken, that there be a foamy spacetime background and an associated preferred class of inertial observers. At some point this author came to ask the question: if a quantum-gravity scenario is shown to lead to a Planck-scale-deformed dispersion relation, does this automatically imply that Lorentz symmetry is broken and there is a preferred class of inertial observers? The answer turned out to be negative. It turned out to be possible to describe kinematics in terms of a new class of relativistic theories, the DSR theories, in which a deformed dispersion relation is implemented by deforming (rather than breaking) Lorentz symmetry and without any preferred class of inertial observers.

Some of the ingredients of a DSR theory are also present in other new-physics scenarios, most notably the intuition that the Planck scale might be one or another kind of fundamental scale. But in DSR the Planck scale is introduced as a specific type of fundamental scale, and the DSR framework prescribes certain connections between the Planck scale and other physical entities. These characteristic features of DSR were summarized in Section 3:

(i) DSR poses a precise condition on the second observer independent scale. It should be a fundamental scale in the sense of c, and <u>not</u> in the sense we presently attribute to $\hbar$.

(ii) Since a key condition is that the second observer-indepedent scale be treatable in complete analogy with what we presently do with c, the main focus of DSR research, the first structure that must be explored in a DSR proposal, is the description of transformation laws between inertial observers. And since the ultimate goal is the description of actual observations/data, one cannot be satisfied with infinitesimal transformations between observers. Boosts of arbitrary magnitude must be considered.

(iii) The second observer-independent scale should become significant in the ultra-

high-energy/ultra-small-wavelength regime.

9.1. *Works on minimum wavelength, maximum acceleration, and similar concepts*

The idea that besides the maximum velicity c and minimum (non-zero) angular momentum $\hbar$ there might be other similar constraints, like a minimum wavelength and a maximum acceleration, has been explored through many views/scenarios in the literature. Although the DSR framework does not necessarily lead to this type of constraint (the second <u>relativistic</u> observer-independent scale can be introduced in other ways), much of the work on DSR has been focusing on the possibility of a maximum-momentum/minimum-wavelength (and maximum energy, in some cases), and in this respect it is naturally connected with this previous literature. The connection however stops at the level of this type of constraint. The DSR requirements that the second observer-independent scale be <u>relativistic</u> (*i.e.* similar to c, not to $\hbar$, see Subsection 3.1), and that this should be reflected in deformed transformation rules between inertial observers, and that the deformation should affect primarily the high-energy regime, were not put in focus before work in the DSR framework.

In Refs. [32,33] deformations of the Heisenberg algebra (phase-space deformations) were explored as a way to obtain the existence of a minimum wavelength. Basically, one would find a minimum wavelength because of a deformed relation between momentum and wavelength. Momentum would still be allowed to go up to infinity, but the relation between momentum and wavelength would be such that infinite momentum would still correspond to a finite minimum value of wavelength. Since in this scenario the minimum wavelength is not the result of deformed transformation laws between inertial observers,[‖] there is at present no connection with DSR research, although it might be intersting to seek a scheme in which this mechanism for minimum wavelength is implemented *a la* DSR. Similar remarks apply to other "minimum wavelength", "minimum length", "minimum length uncertainty" proposals.

Refs. [49,50] elaborated the idea of a maximum proper acceleration (from which a minimum length uncertainty eventually emerged), by considering a phase-space approach to quantum geometry. A deformation of the rules of transformation between inertial observers was not even considered, and, as a consequence, it is difficult to make any assumption about the nature of the maximum-acceleration constraint (should it be *a la* c? or *a la* $\hbar$?). It would be interesting to attempt to implement *a la* DSR this idea of a maximum acceleration.

9.2. *Works on Scale Relativity*

Scale Relativity is a proposal of revision of the principles of relativity that originated in works by Nottale [51] and was further developed through work also by other authors (see, *e.g.*, Refs. [52,53]). It is based on the idea that the relativity principles should involve, in addition to the usual transformation between inertial observers (primarily characterized by the relative orientation of the axes of the observers and the relative velocity of the observers), also a certain type of scale transformations. This is clearly a "beyond special relativity" proposal that is complementary to the DSR proposal. While DSR postulates that the ordinary transformations between observers should be deformed in order to accomodate a second relativistic observer-independent scale, the Scale-Relativity research programme does not necessarily question the form of the ordinary rotations+boosts transformations, but

[‖]The laws of transformation of energy-momentum (and spacetime coordinates) were not even investigated in Refs. [32,33].

rather postulates that in addition to these transformations one should consider an additional class of (scale) transformations.

While as originally formulated (and presently studied) the DSR proposal and the Scale-Relativity proposal are complementary and alternative, it would be interesting to explore the possibility of merging these two proposals (if not in a rigorous sense, at least in the sense of combining some of the ingredients of both). As mentioned in discussing here some of the key open issues for DSR research, the deformed energy-momentum transformations laws of typical DSR schemes could invite us to classify observers according to the energy of the probes they are using, and this might force us to consider a corresponding class of scale transformations.

Similarly, it is not unconcievable that the addition of scale transformations postulated by the Scale-Relativity programme might lead to some consistency requirement concerning the action of rotations and boosts in the energy-momentum sector. The recent interest in the DSR framework should motivate a reanalysis of the Scale-Relativity framework in which the focus be placed on the structure of finite rotation and boost transformations, and such that the possible emergence of an observer-independent scale in the DSR sense could be uncovered. This could lead to a Scale-Relativity extension of the DSR framework, rather than of the ordinary Special-Relativity framework.

9.3. *Works on Fock spacetime*

It has been noticed (already in the paper [5] that proposed DSR2) that the DSR2 energy-momentum transformation rules are formally related to some spacetime transformation rules which were written down long ago by Fock [54]. It should be stressed that Fock was not guided by plans to revise the Special-Relativity postulates (and of course he had no intention of introducing a second observer-independent scale). Fock was rather interested [54] in establishing the precise relation between the conceptual structure of Special Relativity and the nature of the Special-Relativity transformation laws. The type of question Fock was intending to investigate is: if I remove one or another conceptual element of Special Relativity which corresponding generalization of Lorentz spacetime transformations becomes allowed? In one case (removing a corresponding conceptual ingredient of Special Relativity) Fock found a one-parameter family of spacetime transformation laws which turns out to coincide formally with the DSR2 energy-momentum transformation laws.

Fock's motivation was not the one of introducing a second observer-independent scale, and the formulas he eventually stumbled upon require a <u>large distance</u> scale. The corresponding physical effects would be most significant <u>at low energies</u>, contrary to one of the primary conceptual ingredients that characterize the DSR framework (and in clear conflict with everyday observations). The Fock result (which was not even the proposal of a physical theory, it was just an observation on the logical structure of ordinary Special Relativity) is physically completely different from the DSR2 proposal in spite of the amusing similarities of the mathematical formulas. Attributing to Fock the DSR2 proposal of Ref. [5] would be a bit like attributing to Bardeen-Cooper-Schrieffer (and their study of superconductivity) the proposal of the Glashow-Weiberg-Salam Standard Model of particle physics.

9.4. *Works on κ-Poincaré Hopf algebras*

As mentioned, the research programme which has closer connection to the DSR proposal is the one in which quantum gravity was advocated in motivating the emergence of deformed dispersion relations. Before the DSR proposal, authors would automatically assume that these deformed dispersion relations should require that Lorentz symmetry be broken, with the emergence of an associated preferred class of inertial observers. DSR was proposed primarily with the intent/objective of

observing that this might not be true in general: a Planck-scale deformed dispersion relation can be adopted without the support of a preferred class of inertial observers, at the "cost" of a corresponding deformation of the laws of transformation between inertial observers.

Deformed dispersion relations had also been encountered in investigations of the κ-Poincaré Hopf algebras [55,29,20]. The mathematics research line of "quantum groups" had encountered serious resistence in obtaining a quantum deformation of the Poincaré algebra. The κ-Poincaré research line emerged out of the idea of circumventing these difficulties by first achieving a quantum deformation of the symmetry algebra of deSitter space, and then taking a cleaver limiting procedure, which would lead to candidate "quantum" versions of the Poincaré algebra. Many such κ-Poincaré Hopf algebras have been considered [20]. They were not born out of the objective of introducing a second relativistic observer-independent scale, and actually the question about observer-independent scales could not even be properly formulated in the κ-Poincaré framework, since results concerning the exponentiation of Lorentz-like κ-Poincaré algebra elements had been found [29] to have puzzling properties (these exponentiations of algebra elements could not be seen as elements of a group, but only as elements of a "quasigroup" in the sense of Batalin [56]). In a sense, from the technical side, the opportunity for the DSR proposal came out of the realization [1] that these results about problems with the exponentiation of the Lorentz-like elements κ-Poincaré Hopf algebras were not present in all κ-Poincaré Hopf algebras. The results of Ref. [29] had led to the expectation that one would encounter these problems in all κ-Poincaré Hopf algebras, but actually the rotation/boost transformations of the DSR1 proposal can be cast in the framework of a particular κ-Poincaré Hopf algebra (the one called "bicrossproduct basis" in the mathematics literature), and in that particular κ-Poincaré Hopf algebra one encounters no problems concerning the exponentiation of algebra elements, thereby providing a counter-example for the scenario which had emerged in Ref. [29].

The fact that the concept of relativistic transformations between inertial observers was very far from the objectives of work on κ-Poincaré Hopf algebras is also reflected in the structure of the laws of composition of energy-momentum which had been adopted in the relevant literature (see, *e.g.*, Ref. [7] and references therein). According to these laws a particle-producing collision process $a+b \rightarrow c+d$ would lead to a "energy-momentum-conservation" condition of the type $(p_a \dot{+} p_b)^\mu = (p_c \dot{+} p_d)^\mu$, where $\dot{+}$ is a deformed and nonsymmetric (nonabelian sum) law of composition. While the fact that the composition law does not apply symmetrically to the intervening particles had led to some concern, there was no concern (it was not even noticed) about the fact that such laws are not truly covariant under action through the κ-Poincaré algebra elements. Acting with κ-Poincaré algebra elements on $(p_a \dot{+} p_b)^\mu$ on does obtain the transformed $(p'_a \dot{+} p'_b)^\mu$, but unfortunately it is easy to verify [30] that the condition $(p_a \dot{+} p_b)^\mu = (p_c \dot{+} p_d)^\mu$ is incompatible with the condition $(p'_a \dot{+} p'_b)^\mu = (p'_c \dot{+} p'_d)^\mu$. In the new perspective of the DSR proposal one would describe these κ-Poincaré laws as inconsistent with the Relativity Principle. While in certain DSR schems, like DSR1, the one-particle-sector transformation rules can be formulated in terms of (exponentiated) κ-Poincaré boost generators, this problem about composition of momentum actually poses a serious obstacle for the interpretation *a posteriori* (in light of the DSR proposal) of at least some κ-Poincaré algebras as mathematical basis for a DSR physical theory.

Setting aside these problems in the multiparticle sector, it appears proper to perceive the relation between certain DSR theories and work on corresponding κ-Poincaré Hopf algebras in the same way in which we perceive the relation between Einstein's physical theory of Special Relativity and preceding works by FitzGerald, Lorentz and Poincaré. Some κ-Poincaré Hopf algebras provide an important

mathematical background for certain corresponding DSR physical theories. In the DSR theories in which this correspondence is present it may be appropriate to call the transformation laws "κ-Lorentz transfomartion laws", just like we call Lorentz transformations the transformation laws of Einstein's Special Relativity.

10. Outlook

Doubly-Special Relativity is maturing quickly, as a result of the interest it is attracting from various research groups, each bringing its relevant expertise to the programme. Some key results have already been obtained. I have emphasized here the ones that I presently perceive as most significant. Several crucial open issues remain to be tackled. I have attempted to list the most important of these open issues, and I also ventured suggesting possible "lines of attack".

DSR research also clearly must find some additional consistency requirements. For example, it appears plausible that not all nonlinear realizations of the Lorentz group give rise to a physically acceptable DSR theory. But we still lack a suitable consistency requirement that would allow us to identify the cases that must be disregarded. Perhaps some condition of consistency with the emergence of an "acceptable spacetime picture" (whatever that means) should be implemented.

Of course, the most important results that this research line is awaiting are experimental results. We can be moderately optimistic that with new observatories, such as GLAST and the Pierre Auger Observatory, we might have a key experimental hint within a few years. Without this type of experimental guidance work in DSR will remain in the awkward limbo in which Einstein would have been in trying to introduce an observer-independent velocity scale without having available the indication that this observer-independent velocity should be the one of the Maxwell equations and of the Michelson-Morley experiments. We must therefore think hard of other experimental contexts which may be of help for the development (or falsification) of DSR. New opportunities might materialize even on a relatively short time scale. In these closing remarks I want to contemplate an example of such an opportunity which might materialize in the not-so-distant future. I expect that the type of time-of-arrival-difference studies that GLAST will be conducting [35] should somehow be doable even with particles of much higher energies. For example, it is sometimes argued that the mechanism that produces ultra-high-energy cosmic rays is the same mechanism that gives rise to gamma-ray bursts. If gamma-ray bursters also emit (roughly simultaneously with the gamma-ray burst) particles of extremely high energies, we could exploit the fact that, over a time of travel of $10^{17}s$, even with a dispersion relation that is deformed only at the E_p^{-2} level (quadratic Planck-scale suppression), a particle with $E \sim 10^{19}eV$ can acquire a time-arrival difference with respect to particles of $E \sim 10^{6}eV$ (particles which arrive with the main burst) which is of the order of $0.1s$, and possibly observable.

References

[1] G. Amelino-Camelia, gr-qc/0012051, Int. J. Mod. Phys. D11 (2002) 35; hep-th/0012238, Phys. Lett. B510 (2001) 255.
[2] J. Kowalski-Glikman, hep-th/0102098, Phys. Lett. A286 (2001) 391.
[3] G. Amelino-Camelia, gr-qc/0106004 (in "Karpacz 2001, New developments in fundamental interaction theories" pag. 137-150).
[4] N.R. Bruno, G. Amelino-Camelia and J. Kowalski-Glikman, hep-th/0107039, Phys. Lett. B522 (2001) 133.
[5] J. Magueijo and L. Smolin, hep-th/0112090, Phys. Rev. Lett. 88 (2002) 190403.
[6] G. Amelino-Camelia, D. Benedetti, F. D'Andrea, hep-th/0201245.

[7] J. Lukierski and A. Nowicki, hep-th/0203065.
[8] J. Kowalski-Glikman and S. Nowak, hep-th/0204245.
[9] S. Judes, and M. Visser, gr-qc/0205067.
[10] M. Arzano and G. Amelino-Camelia, g-qc/0207003.
[11] D.V. Ahluwalia and M. Kirchbach, gr-qc/0207004.
[12] G. Amelino-Camelia, gr-qc/0207049, Nature 418 (2002) 34.
[13] J. Rembielinski and K.A. Smolinski, hep-th/0207031.
[14] A. Granik, hep-th/0207113.
[15] C. Heuson, "A simple deformation of Special Relativity" (preprint 2002).
[16] J. Magueijo and L. Smolin, gr-qc/0207085.
[17] N.R. Bruno, gr-qc/0207076.
[18] G. Amelino-Camelia, astro-ph/0209232.
[19] A. Feoli, gr-qc/0210038.
[20] J. Lukierski, H. Ruegg and W.J. Zakrzewski, Ann. Phys. 243 (1995) 90.
[21] G. Amelino-Camelia, Phys. Lett. B392 (1997) 283.
[22] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Int. J. Mod. Phys. A12 (1997) 607.
[23] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar, astro-ph/9712103, Nature 393 (1998) 763.
[24] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, hep-th/9207103, Phys. Lett. B293 (1992) 37; hep-th/9503162, Mod. Phys. Lett. A10 (1995) 1685.
[25] R. Gambini and J. Pullin, Phys. Rev. D59 (1999) 124021.
[26] J. Alfaro, H.A. Morales-Tecotl and L.F. Urrutia, Phys. Rev. Lett. 84 (2000) 2318.
[27] H. Sahlmann and T. Thiemann, gr-qc/0207031.
[28] M. R. Douglas and N. A. Nekrasov, hep-th/0106048, Rev. Mod. Phys. 73 (2001) 977.
[29] J. Lukierski, H. Ruegg and W. Ruhl, Phys. Lett. B313 (1993) 357.
[30] G. Amelino-Camelia, gr-qc/0205125.
[31] H.S. Snyder, Phys. Rev. 71 (1947) 38.
[32] A. Kempf, G. Mangano and R.B. Mann, Phys. Rev. D52 (1995) 1108.
[33] D.V. Ahluwalia, Phys. Lett. A275 (2000) 31.
[34] S.D. Biller et al., Phys. Rev. Lett. 83 (1999) 2108.
[35] J.P. Norris, J.T. Bonnell, G.F. Marani and J.D. Scargle, astro-ph/9912136; A. de Angelis, astro-ph/0009271.
[36] T. Kifune, Astrophys. J. Lett. 518 (1999) L21.
[37] H. Sato, astro-ph/0005218.
[38] G. Amelino-Camelia and T. Piran, Phys. Rev. D64 (2001) 036005.
[39] D. Benedetti, Laurea thesis (2002, unpublished).
[40] S. Majid and H. Ruegg, Phys. Lett. B334 (1994) 348.
[41] R.J. Protheroe and H. Meyer, Phys. Lett. B493 (2000) 1.
[42] M. Takeda *et al*, Phys. Rev. Lett. 81 (1998) 1163.
[43] Information on the Pierre Auger Observatory is available at http://www.auger.org/
[44] G. Amelino-Camelia, J.D. Bjorken and S.E. Larsson, *Phys. Rev.* D56 (1997) 6942.
[45] S. Alexander, R. Brandenberger and J. Magueijo, hep-th/0108190.
[46] J.D. Barrow, Phys. Rev. D59 (1999) 043515.
[47] A. Albrecht and J. Magueijo, Phys. Rev. D59 (1999) 043516.
[48] M.A. Clayton and J.W. Moffat, Phys. Lett. B477 (2000) 269.
[49] E.R. Caianiello, Riv. Nuovo Cim. 15 (1992) 1.
[50] E.R. Caianiello, M. Gasperini and G. Scarpetta, Nuovo Cim. 105B (1990) 259.
[51] L. Nottale, Int. J. Mod. Phys. A4 (1989) 5047; Int. J. Mod. Phys. A7 (1992) 4899.
[52] C. Castro, hep-th/9512044, Found. Phys. Lett. 10 (1997) 273

[53] C. Castro and A. Granik, physics/0009088.
[54] V. Fock, "The theory of space-time and gravitation", (Pergamon Press, 1964).
[55] J. Lukierski, A. Nowicki and H. Ruegg, Phys. Lett. B293 (1992) 344.
[56] I.A. Batalin, J. Math. Phys. 22 (1981) 1837.